**Blood pressure (BP):** Blood pressure is the force or pressure of blood on the walls of the blood vessels in which it is contained. Most of this pressure is due to work done by the heart by pumping blood through the [circulatory system](https://en.m.wikipedia.org/wiki/Circulatory_system). Blood pressure is usually expressed in terms of the [systolic pressure](https://en.m.wikipedia.org/wiki/Systole) (maximum during one heartbeat) over [diastolic pressure](https://en.m.wikipedia.org/wiki/Diastole) (minimum in between two [heartbeats](https://en.m.wikipedia.org/wiki/Cardiac_cycle)) and is measured in millimeters of [mercury](https://en.m.wikipedia.org/wiki/Mercury_(element)) ([mmHg](https://en.m.wikipedia.org/wiki/Millimeter_of_mercury)), above the surrounding [atmospheric pressure](https://en.m.wikipedia.org/wiki/Atmospheric_pressure). Normal blood pressure in an [adult](https://en.m.wikipedia.org/wiki/Adult) is approximately 120 mmHg systolic, and 80 mmHg diastolic, abbreviated "120/80 mmHg". Globally, the average blood pressure, age standardized, is approx. 127/79 mmHg in men and 122/77 mmHg in women. Blood pressure is one of the [vital signs](https://en.m.wikipedia.org/wiki/Vital_signs), along with [respiratory rate](https://en.m.wikipedia.org/wiki/Respiratory_rate), [heart rate](https://en.m.wikipedia.org/wiki/Heart_rate), [oxygen saturation](https://en.m.wikipedia.org/wiki/Oxygen_saturation), and [body temperature](https://en.m.wikipedia.org/wiki/Body_temperature).

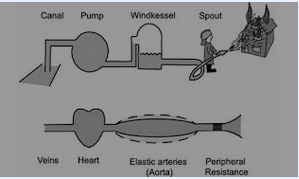
There are two phases of cardiac cycle such as diastolic phase and systolic phase. They occur as the heart beats, pumping blood through a system of blood vessels that carry blood to every part of the body. Systole occurs when the heart contracts to pump blood out and during this phase blood enters the aorta from the heart. Diastole occurs when the heart relaxes after contraction and during this phase the rate of inflow from the heart to the aorta is zero.

**Aorta:** The aorta is the main artery that carries blood away from your heart to the rest of your body. After the blood leaves the heart through the aortic valve, it travels through the aorta, making a cane-shaped curve that connects with other major arteries to deliver oxygen-rich blood to the brain, muscles, and other cells.

**Arterial pulse:** The aorta will be represented by an elastic volume container, whose capacity depends upon the pressure relation. The term “windkessel” may be used for this part of the system. The elastic container is connected to a tube having a definite resistance to flow. This represents the “peripheral resistance”.

Fig.-01: from sheet

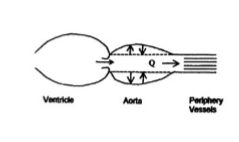
**Windkessel:** Windkessel means an air chamber but it is generally taken to imply an elastic reservoir in large elastic arteries. These arteries distend when the [blood pressure](https://en.wikipedia.org/wiki/Blood_pressure) rises during [systole](https://en.wikipedia.org/wiki/Systole_(medicine)) and recoil when the blood pressure falls during [diastole](https://en.wikipedia.org/wiki/Diastole). Since the rate of blood entering these elastic arteries exceeds that leaving them via the [peripheral resistance](https://en.wikipedia.org/wiki/Peripheral_resistance), there is a net storage of blood in the aorta and large arteries during systole, which discharges during diastole.

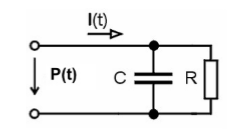


**Windkessel model of blood pressure:** The Windkessel Model was designed by the German physiologist Otto Frank in the late 1800. He described the heart and the systemic arterial system as a closed electric circuit. In his analogy, the circuit contained a water pump connected to a chamber, ﬁlled with water except for a pocket of air. When the pump starts working the water compresses the air, which in turn pushes the water out of the chamber. This analogy resembles the mechanics of the heart. The Windkessel model takes into consideration the following parameters while modeling the cardiac cycle:

* **Arterial Compliance:** refers to the elasticity and extensibility of the major artery during the cardiac cycle.
* **Peripheral Resistance:** refers to the ﬂow resistance encountered by the blood as it ﬂows through the systemic arterial system.
* **Inertia:** simulates the inertia of the blood as it is cycled through the heart.

Arterial Compliance, Peripheral Resistance, and Inertia are modeled as a capacitor, a resistor, and an inductor respectively.



**Figure-02:** arterial blood circulation. 

**Figure-03:** Electrical analog of the 2-element windkessel model.

Here, the current  represents the blood flow from the heart, the electric potential  represents the blood pressure, the capacitance  represents the arterial compliance (or ratio of pressure to volume) and the resistance  represents the peripheral resistance.

It is assumed that the ratio of pressure to volume is constant and that outflow from the Windkessel is proportional to the fluid pressure. Volumetric inflow must equal the sum of the volume stored in the capacitive element and volumetric outflow through the resistive element. This relationship is described by the [differential equation](https://en.wikipedia.org/wiki/Differential_equation):

I ( t ) = P ( t ) R + C d P ( t ) d t {\displaystyle I(t)={P(t) \over R}+C{dP(t) \over dt}} During [diastole](https://en.wikipedia.org/wiki/Diastole) there is no blood inflow since the aortic valve is closed, so.

From (1), we get





Integrating,



where  is an integrating constant.

At  we have , then from (2), we get



Putting the value of  in (2), we get



P ( t ) = P ( t d ) e − ( t − t d ) ( R C ) {\displaystyle P(t)=P(t\_{d})e^{-(t-t\_{d}) \over (RC)}} where is the time of the start of [diastole](https://en.wikipedia.org/wiki/Diastole) and is the blood pressure at the start of diastole. This represents the pressure during diastole.

Again, during systole, blood is ejected into the aorta and can be modeled as a sinusoidal wave, therefore:



where  is time in seconds,  is the period of systole, in seconds and is a constant which represents the maximum amplitude of the blood ﬂow during systole and it’s value is 424 .1 mL.

From (1), we get

This is a first order linear differential equation.

The integrating factor is,



Multiplying (3) by integrating factor we get,





Integrating,



where  is an integrating constant.

At  we have , then from (4), we get



Putting the value of  in (4), we get



This represents the blood pressure during systole.

**Question-01:** Show that during the diastolic period the pressure in the aorta falls in a simple exponential fashon.

OR

Derive the pressure  for diastolic phase.

**Answer:** The linear relationship between the pressure and volume is expressed as,

where,  is a constant,  is the pressure in the windkessel and  is the volume of blood.

If we assume that the pressure at the distal end of the tube (venouside) is zero, then the Poiseuille’s law may written as,

where , here  is the resistance,  is the viscosity of blood,  is the radius and  is the length of the tube.

The minus sign indicates that the greater the pressure  the greater the decrease of volume per unit time in the windkessel.

Eliminating  from equations (1) and (2), we the following differential equation,

Integrating,





where is an integrating constant.

When , then the pressure is .

Using this condition in (4), we get



Putting the value of  in (4), we get

This shows that during the diastolic phase the pressure in the aorta falls in a simple exponential fashon (no flow from the heart).

**Question-02:** Show that the pressure can be found for systolic phase,

 .

**Answer:** The linear relationship between the pressure and volume is expressed as,

where,  is a constant,  is the pressure in the windkessel and  is the volume of blood.

The essential thing about this phase is that the blood is pumped into the aorta (windkessel) by the contraction of the heart. The rate of volume change  in the windkessel is now the resultant of two factors, namely, the rate of inflow ( from the heart) and the rate of outflow ( through the peripheral resistance).

The rate of outflow is given by Poiseuille’s law,

Here omit the minus sign since now we think of  not as volume decrease per unit time in the windkessel, but as the rate of flow i.e, as the volume of fluid passing a cross-section per unit time.

Let us call the volume of fluid entering the windkessel per unit time . It will be clear that the net change of volume per unit time in the windkessel is given by,

If , then  is positive i.e, the volume in the windkessel increases and vice versa.

Now from (3), we get







Putting  in equation (4), we get

Integrating factor is,



Multiplying (5) by integrating factor, we get



Integrating,



where  is an integrating constant.

At  we have .

Using this condition in (7), we get



Putting the value of  in (7), we get

.

This the required pressure in systolic phase.

**Question-03:** What do you mean by a heart function test? Show that the end of work the oxygen debt can be written as .

OR

What do you mean by a heart function test? Obtain the oxygen debt.

**Answer:** **Heart function test:** The measurement of oxygen debt is used as a heart function test. During the exercise, the body is able to incur an oxygen debt. This means that the muscles are able to work without oxygen supply but in the recovery stage oxygen is needed to replenish the energy store during exercise.

If we denote the oxygen debt by  and the work down by , then we have



where  is proportional constant.

Let us divide the equation (1) by time , we get

If we choose a time unit small compared with the duration of the process, we may write (2) by difference quotients,

The change of oxygen debt per second is proportional to the change of work per second.

In applications, we may replace the difference quotient by the derivative,

On the other hand, the oxygen debt also depends upon the oxygen supply. Clearly we may write,

where  denotes the oxygen uptake.

The minus sign is used to indicate the fact the rate of decrease of the oxygen debt is proportional to the rate of increase of the oxygen uptake.

Now, by combining (4) and (5), we get

where  is the net change of oxygen debt per unit time.

We assume that the extra oxygen uptake per second is proportional to the oxygen debt existing at any instant,

Substituting (7) in (6), we obtain

In physics  is called power and given a special name . Hence we may write the equation (8) in the form,





where . This is our basic equation. We shall study various possible function of .

At the moment of exercise is ended, the power become zero. Therefore (9) becomes,





Integrating,



where  is an integrating constant.

When  then .

Using this condition in (10), we get



Putting the value of  in (10), we get

Figure-: from sheet

The equation (11) shows that at the end of work, the oxygen debt  drops to zero in an exponential fashon.

**Question-04:** What do you mean by glucose concentration in blood? Show that it can be expressed as .

OR

Find the time curve of glucose concentration in blood during continuous intravenous injection of glucose.

**Answer:** The Glucose  is a simple sugar that comes from the foods we eat, and it is also formed and stored inside the body. It is the main source of energy for the cells of our body, and it is carried to each cell through the bloodstream. The blood sugar level or the amount of glucose in the blood is called concentration of glucose in the blood. The normal blood glucose level (tested while fasting) for non-diabetics, should be between 3.9 and 7.1 mmol/L (70 to 130 mg/dL). The global mean fasting plasma blood glucose level in human is about 5.5 mmol/L (100 mg/dL); however, this level fluctuates throughout the day. Several hormones, including insulin, control glucose levels in the blood.

If  be the quantity of glucose in the blood and  be the concentration, then the glucose injected is eliminated at a rate proportional to the amount present in the blood.

On the other hand, the glucose concentration in the blood tends to rise as a result of the infusion.

As a result of these two competing process, we finally get

where  is the velocity constant of elimination and  is the volume distribution,  is the concentration and is the rate of infusion.

If no glucose is added i.e, , then the equation (1) reduces to

If there is no elimination i.e, , then the equation (1) reduces to

The equation (1) can be written as

This is a first order linear differential equation.

The integrating factor is,



Multiplying (4) by the integrating factor we get



Integrating,



where  is an integrating constant.

If  when  then from (6), we get



Putting the value of  in (6), we get



This is the time curve of glucose concentration in blood during continuous intravenous injection of glucose. Of course, the glucose concentration on the blood at  is not zero, but the formula (7) remains valid if  represents the glucose concentration in excess of the initial value at the start of the infusion.

**Question-05:** Show that for nervous excitation .

OR

Discuss the Blair’s theory of nervous excitation with experimental evidence.

**Answer:** The mathematical simplicity of Blair’s theory is well suited to serve as an introduction to the theoretical study of nervous excitation.

Let us replace the quantity by the concentration of a so called exciting ion. This assumption is justified by experimental evidence which roughly gives rise to the following picture.

Figure-: from sheet

During electrical stimulation the exciting ion moves towards the cathode so that the concentration of this ion increases near the cathode. Let us assume that the concentration increases at a rate proportional to the applied voltage  and it increases at rate proportional to , where  is the resting concentration of the ion.

Obviously, the net result may be written as,

where  and  are constants.

Let  then



The equation (1) becomes,



Excitation occurs when  reaches a threshold value .





 ; where 

This becomes the condition for excitation.

The equation (2) is a first order linear differential equation.

The integrating factor is,



Multiplying (2) by the integrating factor we get



Integrating,





where  is an integrating constant.

When a suddenly applied constant voltage  is set up at then and equation (4) becomes,



 ; since  

If  becomes very large then from (5), we get



where ,  and  are constants.

But at  is known as the rheobase and denoted by .



From (5), we have

















 ; where .

This is the required result.

**Question-06:** Find the amounts of  in plasma and erythrocytes. Show that  is straight line of slope .

OR

Show that the pressure can be found as .

**Answer:** The basic assumption in this case is that the total amount of trace substance in the system remains constant. The mathematical model is shown in the figure.

Figure-: from sheet

Component represents the plasma containing an amount  of , while  represents the erythrocytes containing an amount  of . On this assumption that the total amount of  within this closed system remains constant, we have

where  denotes a constant and  and are the permeability constants for from A to B and B to A respectively.

The concentration of is constant throughout each of the components A and B. if we consider changes in the component B, we have

The term  represents the contribution to the rate of change of  due to the presence of the amount  in the plasma. On the other hand, the term  represents the amount of  lost per second to the plasma on balance we obtain the equation (2). From (1) and (2), we get





where 

The equation (3) is a first order linear differential equation.

The integrating factor is,



Multiplying (3) by the integrating factor we get



Integrating,



where  is an integrating constant.

If  when , then from equation (5), we get



Putting the value of  in (5), we get



Now, from (1), we have











 . 

When , then  ; .

From (7), we get

From (7) and (8), we have

This is the required solution.

Again, from the equation (9), we get



From (10) it follows that if we plot  against the time  we would obtain a straight line with gradient  and intercept  on -axis.

Figure-: from sheet